INVESTIGATION AND APPLICATION OF REVERBERATION MEASUREMENTS IN WATER

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ABSTRACT

A method of determining acoustic power output of underwater sound sources is presented employing reverberation techniques similar to those employed in air.

Two possible approaches were taken. One following the ideas of R. W. Young, which includes the effect of the direct wave and by use of a correction factor eliminates its effect on the resultant power, which requires knowledge of a steady state average pressure, a decay rate and an average absorption coefficient.

The second follows the idea presented in "Fundamentals of Acoustics" by Kinsler and Frey which requires knowledge of only a steady state average pressure and decay rate.

Both methods yield results consistent to within one decibel of a standard acoustic power as determined by pulse techniques.

A test setup is proposed for use of this technique with comments given as to desirable characteristics of equipment.

The writers wish to express their appreciation for the assistance and encouragement given them in this investigation by Professor L. E. Kinsler of the U. S. Naval Postgraduate School and also to Miss Delores A. Price of U. S. Navy Electronics Laboratory for assistance in calibration of LC-32 hydrophones used.

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SYMBOL TABLE

a	Sabine absorption in Sabins, equals $S\bar{a}$
$\bar{a}$	average Sabine coefficient for a room
a_i	random incidence energy absorption coefficient of a particular surface of area S_i
$\bar{\alpha}$	average random incidence energy absorption coefficient
c	speed of sound in the medium in the room or enclosure
D	decay rate of sound pressure level db/sec
$\mathcal{E}$	average acoustic energy density equals $P^2/\rho c^2$
E	RMS voltage
f_o	center frequency of noise band
I	acoustic intensity
K	constant
P	acoustic pressure(rms)
ρ	density of the medium
R_p	equivalent parallel resistance of hydrophone
S	total surface area of an enclosure
S_i	area of a specific side of the enclosure
T	reverberation time in seconds equals 60/D
V	volume of the enclosure
W	acoustic power output of the source in watts

1. Introduction.

The determination of absorption coefficients of acoustical materials has attracted considerable interest and effort over the years.

Recently R. W. Young[1] published a review article in this field which proposed modification to present engineering practices. The paper also presented a method for determining the sound power output of a transducer by measurement of decay rate, a steady state sound pressure and an average absorption coefficient in the enclosure under consideration.

R. W. Case and E. W. Vahlkamp[2] made measurements of power through decay rates as proposed by Young but were unable to achieve consistent agreement with power measured by pulse techniques.

Kinsler and Frey[3] also suggest a second or alternative equation for the calculation of acoustic power from measurement of decay rate and a steady state pressure.

The purpose of this investigation is to test the validity of these theories on power measurements as applied to a reverberation tank and a water medium.

[1] R. W. Young, Sabine Reverberation Equation and Sound Power Calculations, J.A.S.A., 31, pp 912-921 July 1959

[2] R. W. Case and E. W. Vahlkamp, Investigation and Application of Reverberation Measurements in Water, USNPGS, Thesis, 1962

[3] L. E. Kinsler and A. R. Frey, Fundamentals of Acoustics Second Edition, John Wiley & Sons, 1962

2. Theory.

All the contributions to architectural acoustics before the work of W. C. Sabine were of a qualitative nature. Sabine was the first to commence a comprehensive and quantitative study of the subject and, since science and engineering are basically quantitative, it may be fairly stated that the science of architectural acoustics had its beginning with Sabine. In his paper in 1900[4], Sabine set forth in simple, but comprehensive, language the requirements for good hearing in any auditorium as follows:

> It is necessary that the sound should be sufficiently loud; that the simultaneous components of a complex sound should maintain their proper relative intensities; and that the successive sounds in rapidly moving articulation, either speech or music, should be clear and distinct, free from each other and from extraneous noises. These three are the necessary, as they are the entirely sufficient, conditions for good hearing.

It seemed apparent to Sabine that the third of these factors was the most important one in affecting the acoustical quality of a room and he therefore devoted a good number of years to a quantitative study of the growth and decay of sound in an enclosure. From this study he formulated the following empirical equation:

$$T = \frac{KV}{\bar{a}S} \qquad (1)$$

[4] W. C. Sabine, Collected Papers on Acoustics, Harvard University Press, 1922

where: T = reverberation time
k = constant
V = volume of the enclosure
S = total surface area of the enclosure
$\bar{a}$ = average Sabine absorption coefficient of the enclosure

Several authors[5,6,7] set forth a derivation following the method developed by Jaeger[8] based on the following assumptions:

1. A random or diffuse distribution of the flow of sound in the room is required and 2. a continuous absorption of sound by the boundaries of the room occurs. These conditions are quite completely fulfilled in highly reverberant rooms. It is to be expected therefore that these formulas would be satisfactory for the practical calculations of reverberation in live rooms but that they would lead to only approximate results in dead rooms. To say a room is "live" we mean that the absorptive power is small, the rate of decay is slow and hence the reverberation time is long.

The assumptions above give rise to the following differential equation with regard to the conservation of energy:

$$\begin{bmatrix}\text{Rate of increase}\\ \text{of energy in room}\end{bmatrix} = \begin{bmatrix}\text{Rate of emission}\\ \text{of energy from}\\ \text{source}\end{bmatrix} - \begin{bmatrix}\text{Rate of absorption}\\ \text{of energy by walls}\end{bmatrix} \qquad (2)$$

[5] V. O. Knudsen, Architectural Acoustics, John Wiley and Sons, Inc., 1932

[6] P. M. Morse and R. H. Bolt, Sound Waves in Rooms, Rev. Mod. Phys., 16, p 76, Jan 1944

[7] L. E. Kinsler and A. R. Frey, op. cit.

[8] A. Jaeger, Zur Theorie des Nachalls, Akad, Wiss. Wein 2A, 120, p 613, 1911

Let $\mathcal{E}$ be the average acoustic energy density, V the volume of the enclosure, and W the power output of the source. Then the rate of absorption of energy by the walls can be shown to be

$$\mathcal{E} c a/4 \tag{3}$$

where c is the speed of sound and a is the total absorptive power of a live room which is found from

$$a = \Sigma a_i S_i \tag{4}$$

where a_i are the respective absorption coefficients representing the fraction of randomly incident energy absorbed by the different materials of areas S_i forming the interior walls of the room as well as any other absorbing surfaces.

The rate at which the energy increases in the room is given by

$$V \frac{d\mathcal{E}}{dt} \tag{5}$$

Substitution of equations (3) and (5) into (2) yields

$$V \frac{d\mathcal{E}}{dt} = W - \frac{\mathcal{E} c a}{4} \tag{6}$$

Assuming the sound source to have been started at t = 0 the solution of equation (6) is

$$\mathcal{E} = \frac{4W}{ac}\left(1 - e^{-\frac{act}{4V}}\right) \tag{7}$$

This can be written in terms of the mean square acoustic pressure by use of the relation that

$$\mathcal{E} = \frac{p^2}{\rho c^2} \tag{8}$$

This results in

$$p^2 = \frac{4W\rho c}{a}\left(1 - e^{-\frac{act}{4V}}\right) \tag{9}$$

If the source is left on until steady state is reached, i.e. $t = \infty$ equation (9) reduces to

$$P^2 = \frac{4W\rho c}{a} \tag{10}$$

or

$$W = \frac{P^2 a}{4\rho c} \tag{11}$$

The differential equation governing the decay of uniformly distributed diffuse sound in a live room is obtained by letting W = 0 in equation (6). If we assume the source is turned off at t = 0 and letting $\mathcal{E}_0$ represent the assumed uniformly distributed energy density at this instant then as t increases

$$\mathcal{E} = \mathcal{E}_0 e^{-\frac{act}{4V}} \tag{12}$$

or in terms of intensity

$$I/I_0 = e^{-\frac{act}{4V}} \tag{13}$$

Applying the operator 10log to both sides of the equation it becomes

$$\Delta IL = 10\log e^{-\frac{act}{4V}} = -\frac{1.087\ act}{V}$$

where $\Delta IL = 10\log I/I_o$ represents the change in intensity level in decibels. It can now be said that the intensity level in a live room and correspondingly the sound pressure level decreases with elapsed time at a constant decay rate D in decibels per second given by

$$D = \frac{1.087\ ac}{V} \tag{14}$$

In accordance with the original idea of Sabine, let us define reverberation time T as the time required for the level of the sound

in the room to decay by 60 db. Then

$$T = \frac{60}{D} = \frac{55.2\,V}{\bar{a}\,c} \tag{15}$$

which is the form of Sabine's emperical equation.

It can be seen however that under dead room conditions, i.e. $\bar{a}$ very large, approaching unity, the predicted reverberation time by equation (15) is

$$T = \frac{55.2\,V}{S\,c} \tag{16}$$

which is not true since we know that for a perfect absorber the reverberation time is zero.

Eyring[9] produced the first important modification to Sabine's equation. His derivation was based on image sources, all of which came into existence when the real source starts. The growth of the intensity is then an accumulation of successive increments from the true source, from the first order or single reflection images, from second order or double reflection images and so on until all sources of any appreciable strength have made their contribution. At this time steady state is reached and the rate of absorption and rate of emission of sound are equal. Similarly when the true source is stopped, all the images are stopped. The decay in energy density in the room therefore results from the successive losses of acoustic radiation; first from the true source, then from the first image and so on until all the images have radiated their energy into the room. From this we see that the duration of audibility is equal to the time required for sound to arrive at the room from the order of images which

[9] C. F. Eyring, Reverberation Time in "Dead" Rooms, J.A.S.A., 1, pp 217-241, Jan. 1930

is so far removed from the room that all images beyond this order will contribute enough energy to be 60 db down from the steady state energy.

It can be seen that the Sabine assumption of uniform distribution of energy and random flow of energy are fulfilled but the decay is not continuous but occurs in steps resulting in a greater absorption during the same interval of time and a more rapid decay rate. This is the essential difference in Eyring's equation and when used in dead rooms gives results superior to the Sabine equation.

Norris[10] suggests a method for deducing Eyring's equation with the result being the following equation:

$$T = \frac{KV}{-S \ln(1-\bar{\alpha})} \tag{17}$$

or in terms of decay rate

$$D = \frac{-1.087\, cS \ln(1-\bar{\alpha})}{V} \tag{18}$$

where

$$\bar{\alpha} = (\Sigma\, a_i S_i)/\Sigma\, S_i \tag{19}$$

It should be noted the Eyring equation uses an "average absorption coefficient". It is assumed that the walls are uniformly covered with material of uniform absorption or that the material is sufficiently well spread over the walls so that an average value can be taken.

When $\bar{\alpha}$ is small it can be seen that Eyring's equation reduces to Sabine's equation which is to be expected for a live room. However, it should be noted that the Eyring form of the equation is seriously in error for very nonuniformly placed absorbents. An

[10] R. F. Norris, A Derivation of the Reverberation Formula, Appendix II in Architectural Acoustics by V. O. Knudsen, John Wiley and Sons, 1932.

example of this would be an enclosure in which one wall is highly absorbent and the others are highly reflective.

Millington and Sette[11,12] proposed another form of averaging the absorption coefficient. Their equation is:

$$T = \frac{KV}{-\sum S_i \ln(1 - a_i)} \tag{20}$$

The main point of difference is this: Norris - Eyring's theory assumes that the energy in the room resumes uniform distribution after each set of incidences during the discontinuous decay processes; Millington and Sette follow the course of a bundle of rays through many reflections and assume that, on the average, a particular ray will strike a given surface a number of times proportional to its area. Both forms assume the Sabine geometric conditions but the averaging is obtained differently. Millington and Sette take a geometric mean whereas Norris - Eyring's is an arithmetic mean. The serious defect of the Millington and Sette form is that it predicts T = 0 if any surface, no matter how small in area, is a perfect absorber.

Fitzroy[13] proposes another modification to Eyring's equation which yields reasonable results for rooms in which the absorption coefficients differ widely. This formula is different as follows:

[11] G. Millington, J.A.S.A. 4, pp 69-82, 1932

[12] W. J. Sette, J.A.S.A. 4, pp 193-210, 1933

[13] D. Fitzroy, J.A.S.A. 31, pp 893-897, 1959

in the case of a rectangular room, three different calculations are made by means of the Eyring equation but the average absorption is changed for each pair of boundaries. Ratios are established relating each pair of boundary areas to the total room area.

This equation can be written thus:

$$T = \frac{x}{S}\left[\frac{KV}{-S\ \ln(1-a_x)}\right] + \frac{y}{S}\left[\frac{KV}{-S\ \ln(1-a_y)}\right] + \frac{z}{S}\left[\frac{KV}{-S\ \ln(1-a_z)}\right] \qquad (21)$$

Where:
- x = total side wall area
- y = total ceiling and floor area
- z = total end wall area
- S = $x+y+z$
- a_x = average absorptivity in x area
- a_y = average absorptivity in y area
- a_z = average absorptivity in z area

It should be noted here that care must be taken to determine whether the room is live or dead and to determine which equation will best fit the given geometry. To quote Knudsen[14],

> The approximate theories, when used with caution and understanding, have served satisfactorily for practical purposes of acoustical designing and they will continue to do so until they are superseded by more exact theories.

The application of room acoustics to a water medium seems to be the solution to one of the problems that has arisen in underwater measurements.

[14] V. O. Knudson, Recent Developments in Architectural Acoustics, Rev. Mod. Phys, 6, p 3, Jan 1934

Test tanks, which have simulated free field conditions for small, high frequency transducers of the past, are no longer good free field approximations for the large, low frequency transducers of today. Other problems have also arisen. The solution or partial solution to these problems seems to lie in the use of a reverberation chamber.

In Kinsler and Frey[15] an equation is given for measuring acoustic power output of sound sources in a reverberant chamber.

If we take equation (11) and substitute for a from equation (14) the following results:

$$W = \frac{P^2 V D}{4(1.087)\rho c^2} \tag{22}$$

It is interesting to note that this equation is independent of a and does not require knowledge of the absorption coefficients of the room as long as it is live. If we rewrite our differential equation (6)

$$V\frac{d\mathcal{E}}{dt} + \frac{ac\mathcal{E}}{4} = W \tag{23}$$

and substitute $a = -S\ln(1-\bar{\alpha})$ after Eyring, we find the solution to be

$$\mathcal{E} = \frac{4W}{-cS\ln(1-\bar{\alpha})}\left[1 - \exp\left(\frac{cS\ln(1-\bar{\alpha})t}{4V}\right)\right] \tag{24}$$

or in terms of average pressure

$$\frac{P^2}{\rho c^2} = \frac{4W}{-cS\ln(1-\bar{\alpha})}\left[1 - \exp\left(\frac{cS\ln(1-\bar{\alpha})t}{4V}\right)\right] \tag{25}$$

[15] Kinsler and Frey, op.cit., pp 436

at steady state or $t = \infty$, we find

$$W = \frac{-cS\ln(1-\bar{\alpha})P^2}{4\rho c^2} \tag{26}$$

substituting equation (18) for decay rate we find

$$W = \frac{P^2 V D}{4(1.087)\rho c^2} \tag{27}$$

Which is identical to equation (22) above.

Young[16] presents several different forms of power equations to obtain power output of a transducer by measurement of decay rate, steady state sound pressure and an average absorption coefficient. One of the forms presented suitable for use in a reverberant chamber is

$$W = \frac{P^2 V D}{4(1.087)\rho c^2}\left[\frac{1-e^{-\bar{a}}}{\bar{a}}\right] \tag{28}$$

This equation is derived from

$$W = \frac{P^2 S \bar{\alpha}}{4\rho c} \tag{29}$$

which has been arrived at not by ignoring the direct sound, but by adding it to the reflected sound. By substitution of

$$\bar{\alpha} = 1 - e^{-\bar{a}} \tag{30}$$

and

$$S = \frac{VD}{1.087\, c\, \bar{a}} \tag{31}$$

equation (28) above results.

It should be noted that for small $\bar{a}$ this expression reduces to equation (22).

[16] R. W. Young, op. cit., pp 916-918.

3. Description of tank and lining material.

A small free standing tank (1.82 x 1.22 x variable depth to a maximum of 0.75 meters) was used in making the reverberation measurements. It was constructed from 0.004 meter thick black iron sheet and set upon 0.02 meter thick rubber pads resting on 0.15 meter wooden blocks.

An unlined tank, its interior painted with a glossy enamel, was first investigated. The second condition under which the reverberation phenomenon was studied was with a few blocks of cone lattice structure aluminum loaded butyl rubber lining[17]. These blocks are 0.2 x 0.41 meter and were used in varying numbers from one to twelve which resulted in a variable decay rate.

It should be noted that the maximum decay observed with 12 blocks at 50Kc results in an $\bar{a}$ of 0.165.

[17] A. Heller, Underwater Anechoic Tank Linings, Navord Report 2989 (Confidential), U. S. Naval Ordnance Laboratory, White Oak, Maryland, pp 19-22, 10 Nov 1953

4. Description of equipment and technique.

Figure 1 shows the block diagram of the equipment used in reverberation measurements. The low level noise signal was passed through a bandpass filter having the following characteristics; lower 3db point at $.78f_o$, upper 3db point at $1.28f_o$ and 18db per octave slope. The General Radio 1391 Pulse and Time Delay Generator was triggered by the square wave generator which controlled the repetition rate of the noise pulses. When repetition rates lower than one per second were required, a battery and hand key were substituted for the square wave generator. In this way the noise signal could be turned on and off at a rate adjustable to allow full buildup and decay of the sound field. The filtered noise signal was then amplified to a value of 3.3 volts RMS which was maintained throughout the investigation and transmitted into the water by an LC-32 hydrophone. The receiving portion of the setup consisted of a similar LC-32 hydrophone followed by a linear amplifier, logarithmic amplifier and Memoscope. For low decay rates, up to a maximum of approximately 200 db/sec, the Brüel and Kjaer recorder was useful.

The Memoscope vertical scale was calibrated in db to facilitate reading the decay rate in db/sec. The storage facility of the Memoscope permitted retracing the pattern and then by use of a Polaroid camera a permanent record was made.

The same equipment was used to make steady state pressure measurements employing a continuous signal and the Fluke Model 910A true RMS voltmeter.

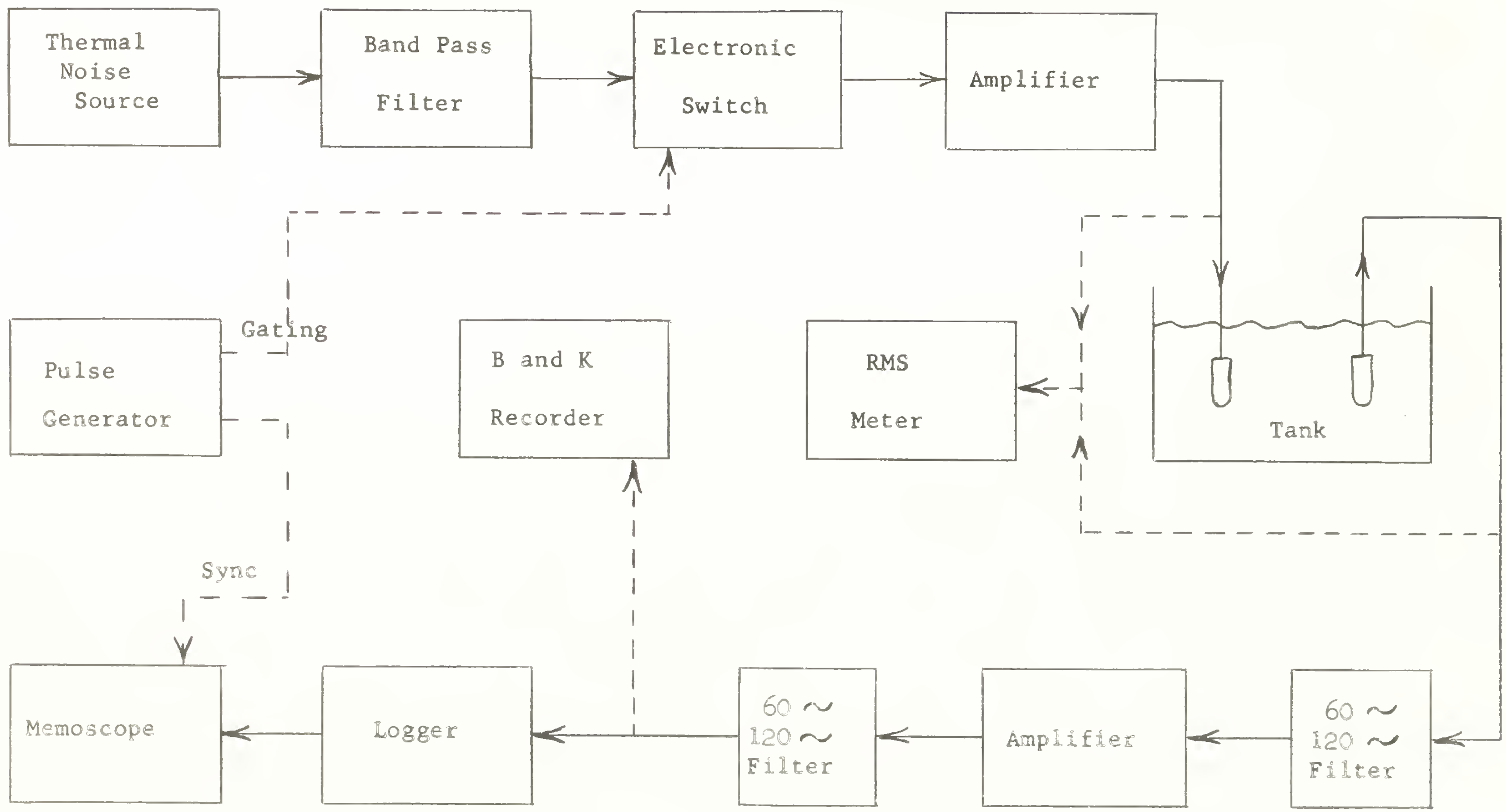


Figure 1 Block Diagram of Equipment Used for Reverberation Measurements

The equipment for pulse technique measurements is essentially the same as Figure 1 with the substitution of a Western Electric 17B oscillator in place of the noise source and filter. Measurement of input and output voltages were made by suitably calibrating the Memoscope with a known voltage.

The filtered noise signal resulted in a considerably diminished standing wave ratio of approximately 2db as compared to approximately 30db for a sine wave.

It should be noted that the tank linings were not uniform but the $\bar{a}$ is such that the tank is live, i.e. $\bar{a} < 0.2$ and Sabine's equation and theory applies. The actual value of $\bar{a}$ is not important to the calculations since decay rates are used.

The testing procedure consisted of taking reverberation measurements with no lining, six blocks lining and twelve blocks lining with the addition of a balloon as noted below. These conditions were repeated over a period of several days which required removal of the blocks from the tank in order to start a new series of measurements. The initial placement of the blocks was in a random manner. Successive tests were made with the blocks placed in the same approximate position so as to minimize the effect on decay rate by variations in placement of the sound absorbing material. It is thought that this step is really not necessary since $\bar{a}$ is < 0.2 but this thought was not fully investigated.

Experiments were made with various diffusers to eliminate or reduce the direct wave. The end result was that a balloon filled

with air and weighted to hold it in place served best. When placed in line between the transducers the amplitude of the direct wave was reduced one order of magnitude. The balloon adds absorption and the result is an increase in decay rate from 97 db/sec. in the unlined tank without the balloon to 192 db/sec. in the unlined tank with balloon.

As more absorptive material is added the reverberant field becomes less intense and less diffuse, resulting in a larger standing wave ratio. The balloon also serves to break up this standing wave. For this reason the balloon was left in the tank when investigating Young's equation, but was moved out of the path of the direct wave.

The balloon could be described as being the shape of an inverted pear with a length of 25 cm and a diameter of 12 cm.

According to Allred and Newhouse[18] there is a slight disagreement between the mean free path obtained by use of the equation 4V/S and the value obtained by Monte Carlo methods. In the situation under study it is noted that our tank 1.8X1.2X0.5 can be scaled to 10X6.66X2.78. By comparing this with Allred and Newhouses 10X6X2 we see the correction factor is 1.032 or a 3.2% error is made. This is considered negligible in the case under study but for work of the most precise nature should be considered.

[18]Allred and Newhouse, Applications of the Monte Carlo Method to Architectural Acoustics, J.A.S.A., 30, pp. 903-904, Oct., 1958.

5. Results.

From the theory presented previously we must formulate working equations to take into account the directivity and receiving response of the LC-32 hydrophones used.

The directivity ratio of a hydrophone is given by

$$D = \frac{I_{ref}}{I_{ave}} = \frac{P_{ref}^2}{P_{ave}^2} \quad (32)$$

where: D = directivity ratio subscripted with a t or r to indicate whether transmitting or receiving directivity.

I_{ref} = the intensity along the reference axis at some distance r from the source.

I_{ave} = the average intensity defined by $I_{ave} = W/4\pi r^2$ where W is the total acoustic power radiated by the source.

The same notation applies to P, the RMS pressure derived from the relation $I = P^2/\rho c$.

By pulse technique it is possible to measure a standard acoustic power output and from this, knowing the input voltage and equivalent parallel resistance of the hydrophone, the efficiency can be calculated. The resultant power output and efficiencies are then compared with those observed by reverberation technique.

Pulse technique power is given by

$$W = \frac{4\pi r^2 P_{ave}^2}{\rho c} \quad (33)$$

In the method used a voltage proportional to the axial pressure is measured. Hence the average pressure is found by

$$P_{AVE}^2 = \frac{E^2}{M^2 D_t} \quad (34)$$

where E is the RMS voltage measured
M is the hydrophone receiving response in volts per newtons/square meter = E/P

Thus our working equation becomes

$$W = \frac{4\pi r^2 E^2}{\rho c M^2 D_t} \tag{35}$$

where D_t is the transmitting directivity of the LC-32 hydrophone

In the reverberation technique the power is given by equation (22)

$$W = \frac{P_{ave}^2 D V}{4(1.087)\rho c^2} \tag{36}$$

In this technique the voltage measured is proportional to the average pressure but since the response of the LC-32 hydrophone is given with respect to the axial pressure, the tollowing expression results

$$W = \frac{E^2 D_r D V}{M^2\, 4(1.087)\rho c^2} \tag{37}$$

A discussion of directivity ratios is given in Appendix I.

Sample Calculations:

A. Pulse Power at 30Kc using equation (35).

$$W = \frac{4\pi r^2 E^2}{\rho c M^2 D_t}$$

r = .316 meter
ρ = 998 Kg/m^3
c = 1485.8 m/s (at 21.5°C)
M_{db} = -105.08 db 1/M^2 = 3.215×10^8 Newton^2/meter^4 volt^2
D_t = 1.63
E = 1.48×10^{-3} volts RMS

$$W = \frac{4\pi(.316)^2(3.215\times10^8)(1.48\times10^{-3})^2}{(998)(1485.8)(1.63)}$$

$= .364$ milliwatts

B. Reverberation Power at 30 Kc using equation (37) no lining, no balloon.

$$W = \frac{E^2 D V D_r}{M^2\, 4(1.087)\rho c^2}$$

$V = 1.08$ meter^3
$D_r = 1.60$
$D = 97$ db/sec
$E = 8.45 \times 10^{-3}$ volts RMS

$$W = \frac{(1.08)(1.60)(3.215\times10^8)(97)(8.45\times10^{-3})}{4(1.087)(998)(1485.8)^2}$$

$= .403$ milliwatts

C. Reverberation Power at 30 Kc using equation (28) modified, 12 blocks, balloon not in path of direct wave.

$D = 1675$ db/sec.
$S = 7.32$ meter^2
$E = 2.14 \times 10^{-3}$ volts RMS

from equation (14)

$$\bar{a} = \frac{DV}{1.087\, Sc}$$

$$\bar{a} = \frac{1675(1.08)}{1.087(7.32)(1485.8)} = .154$$

$$W = \frac{E^2 D V D_r}{M^2\, 4(1.087)\rho c^2}\left[\frac{1 - e^{-\bar{a}}}{\bar{a}}\right]$$

$$= \frac{(2.14\times 10^{-3})^2 (1675)(1.08)(1.60)(3.215\times 10^{8})}{4(1.087)(998)(1485.8)^2}\left[\frac{1 - e^{-.154}}{.154}\right]$$

$= .415$ milliwatts

Comparison of these, i.e. Reverberation power against Pulse power is not readily obvious due to the difference in true input power for the same voltage input. It is not to be expected that the loading of the medium will be the same since in the case of the pulse technique the transducer sees essentially a free field, but sees a standing wave pattern under reverberation conditions.

Measurements of R_p were made under these conditions with the results being shown in Table II-2, Appendix II.

We can best compare the power by "normalizing" the pulse power to an equivalent reverberation power. The procedure is as follows:

Let W_p = the output acoustic power measured under pulse technique with an input voltage E_1 volts, transducer equivalent parallel resistance R_{p1} and efficiency η_1. Then we can say

$$W_p = \frac{E_1^2 \eta_1}{R_{p1}}$$

Similarly for the reverberation case let W_R = the measured acoustic power, E_2 the input voltage (held constant, $E_2 = E_1$), R_{p2} the parallel resistance and η_2 the efficiency. Then

$$W_R = \frac{E_2^2 \eta_2}{R_{p2}} = \frac{E_1^2 \eta_2}{R_{p2}}$$

Since we are considering the same transducer at the same center frequency $\eta_1 = \eta_2$. Thus

$$W_R = \frac{R_{p1} W_P}{R_{p2}}$$

In our case at 30 Kc we find

$R_{p1} = 6.55$ K ohms

$R_{p2} = 5.76$ K ohms

Thus we can say that the power measured by pulse technique being 0.364 milliwatts should be normalized to

$$.364 \times \frac{6.55}{5.76} = .414 \text{ milliwatts}$$

Let us call the new value the standard or true value of the power.

Now we can compare our reverberation power to the standard value. Table 1 is such a comparison.

Figures 2 thru 8 show typical decay curves for 30KC under the various conditions studied.

A comment should be made about the notation used in Table 1 and in the figures. "Balloon in path" means the balloon was placed between the transducers blocking the path of the direct wave and reducing its amplitude by approximately one order of magnitude.

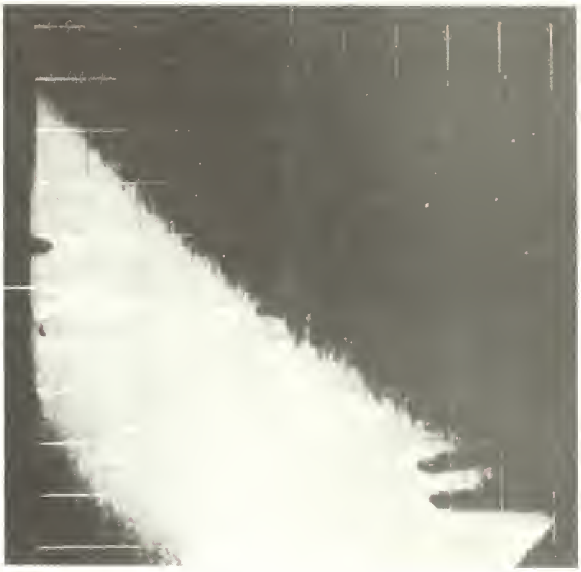
Equation (37) is used for these calculations. "Balloon not in path" means the balloon was in the tank but not between the transducers in the path of the direct wave. Equation (28), modified as shown in sample calculation C, is used for these calculations.

Table 1

Acoustic Power Output (milliwatts) for 3.3 V RMS Input

Center Frequency Kilocycles	10	15	20	25	30	35	40	45	50
Pulse Tech. Measured	.0102	.0381	.105	.1775	.364	.591	1.01	1.88	2.98
Pulse Tech. Standard[1]	.0123	.0474	.131	.221	.414	.669	1.14	2.02	3.12
No lining No balloon	.0127	.0458	.117	.195	.412	.684	1.05	2.05	3.00
No lining, balloon in path	.0138	.0495	.115	.194	.439	.763	1.18	2.37	3.32
No lining, balloon not in path	.0128	.0558	.149	.256	.512	.835	1.28	2.38	3.61
6 blocks, balloon in path	.0118	.0417	.121	.199	.394	.615	1.05	1.90	2.88
6 blocks, balloon not in path	.0110	.0446	.131	.211	.476	.800	1.21	2.20	3.00
12 blocks, balloon in path	.0106	.0451	.127	.198	.408	.642	1.04	2.09	2.89
12 blocks, balloon not in path	.0129	.0481	.110	.194	.425	.667	1.04	2.02	3.03

[1] This is pulse power when "normalized" for differences in R_p as noted in text.



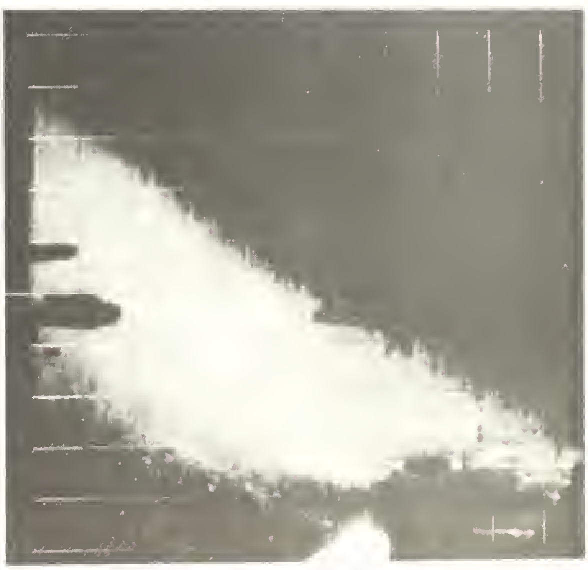
Vertical Scale:
5 db/division

Horizontal Scale:
50 msec/division

E = 8.45 milliv[illegible] RMS
D = 97 db/sec
W = .414 milliwa[illegible]

Figure [illegible]

Photograph of Decay Rate; No lining; No Balloon, 30Kc



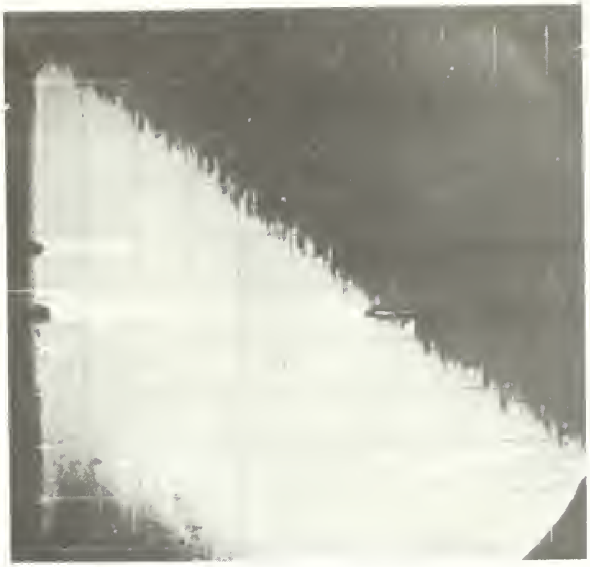
Vertical Scale:
5 db/division

Horizontal Scale:
20 msec/divis[illegible]

E = [illegible].2 millivolts RMS
D = 192.5 db/sec
W = .440 milliwatts

Figure [illegible]

Photograph of Decay [illegible] [illegible], with Balloon in Path, [illegible]Kc



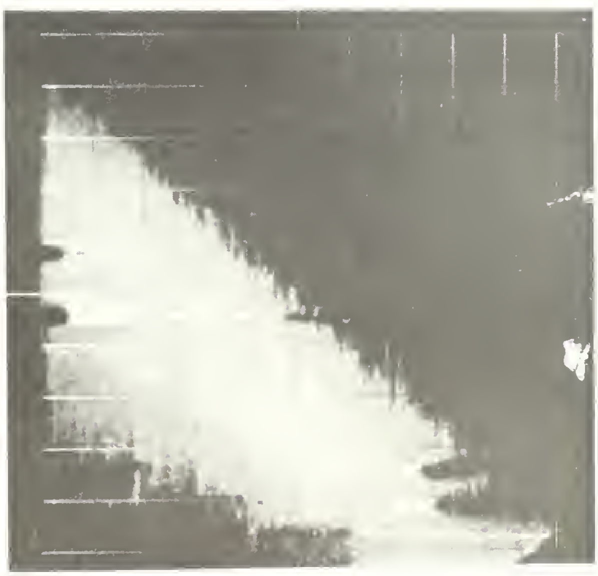
Vertical Scale:
5 db/division

Horizontal Scale:
20 msec/division

E = 6.66 millivolts RMS
D = 192 db/sec
W = .515 milliwatts

Figure 4

Photograph of Decay Rate; No Lining, Balloon not in Path, 30Kc



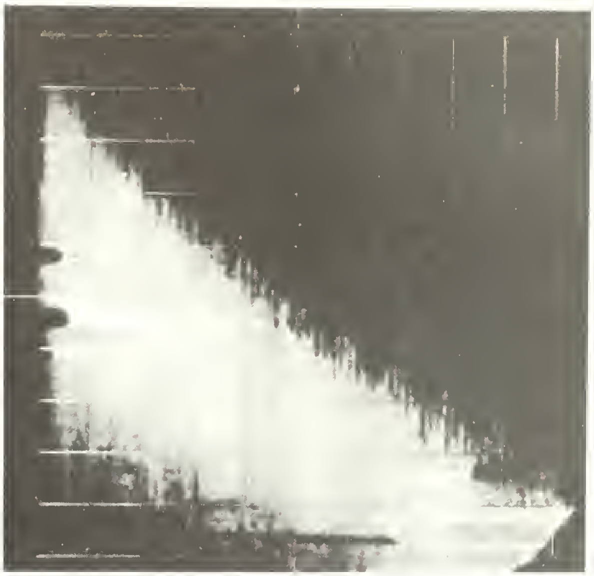
Vertical Scale:
5 db/division

Horizontal Scale:
5 msec/division

E = 2.65 millivolts RMS
D = 940 db/sec
W = .394 milliwatts

Figure 5

Photograph of Decay Rate; Six Blocks of Lining, With Balloon in Path, 30Kc



Vertical Scale:
5 db/division

Horizontal Scale:
5 msec/division

E = 2.95 millivolts RMS
D = 950 db/sec
W = .472 milliwatts

Figure 6

Photograph of Decay Rate; Six Blocks of Lining, Balloon not in Path, 30Kc

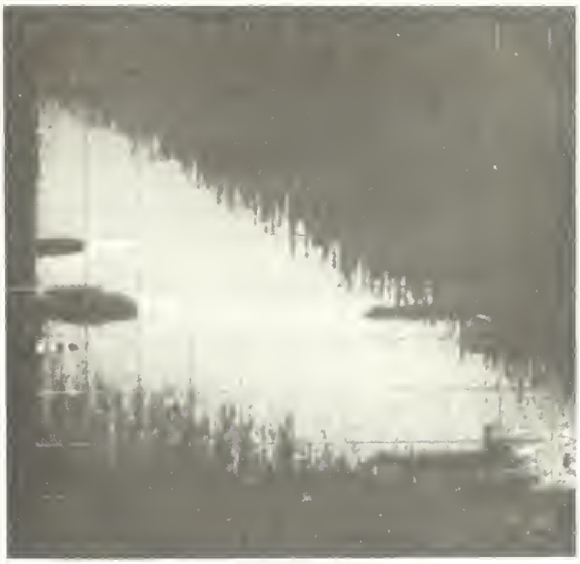
Vertical Scale:
5 db/division

Horizontal Scale:
2 msec/division

E = 1.98 millivolts RMS
D = 1750 db/sec
W = .408 milliwatts

Figure 7

Photograph of Decay Rate; 12 Blocks of Lining, With Balloon in Path, 30Kc



Vertical Scale:
5 db/division

Horizontal Scale:
2 msec/division

E = 2.15 millivolts RMS
D = 1650 db/sec
W = .425 milliwatts

Figure 8

Photograph of Decay Rate; 12 Blocks of Lining, Balloon not in Path
30Kc

6. Conclusions.

From the results it can be seen that under conditions of equal efficiency the powers measured are well within one decibel. Thus the reverberation technique could be used to measure acoustic power output with reasonable accuracy.

The reverberation technique appears to have certain advantages over the pulse technique or other techniques. These are:

1. Relatively simple and quick.

2. The output voltage can be measured directly by use of a true rms meter and does not require complicated gating setups as required in some pulse setups.

3. Gives results within 1 db of pulse powers which in themselves have about 1 db accuracy.

The disadvantages of such a system are:

1. The slope of the decay curve must be evaluated.

2. The source of the noise power must be controllable, i.e. it must be capable of being shut off when desired. If the source is not controllable, a secondary source with the same frequency can be used to determine the decay rate for the enclosure. The source under test can then be used to determine the average steady state pressure.

One possible situation that arises is in the measurement of machinery noise output of a submarine or surface ship in an enclosure such as a drydock. By measuring an average pressure, either by rotating the hydrophone or by measurements in varied locations,

and measuring the reverberation time equation (22) can be used to calculate the power. This, of course, includes the direct wave. Equation (28) can be used if an average absorption coefficient is calculated. It should be noted that the difference between the power calculated by equation (28) is different from that calculated by equation (22) by the factor $(1-2^{-\bar{a}})/\bar{a}$ which for $\bar{a}$ 0.2 results in less than 10% error. Thus particular care need not be taken to ensure elimination of the direct wave, provided the lest tank is live.

This technique could also be used in determining the effect of noise reduction treatment to a piece of machinery for ship quieting. In a given tank and at a specific frequency the $\bar{a}$ would be a constant and the measurements taken before and after treatment would contain the same error and a true measure of the effectiveness of the treatment would be given.

A few comments should be made as to desirable characteristics of the equipment used. First an omnidirectional hydrophone at the frequencies under consideration is highly desirable. This eliminates the problem of orienting the hydrophone and taking into account its directivity ratio.

If a completely omnidirectional hydrophone is not available, one that has unity directivity ratio in the horizontal plane is desirable. This is so as to correctly account for the direct wave since in the proposed uses of the technique the direct wave will still be present.

If the hydrophone is non-directional in the horizontal plane care need not be taken in its orientation. Otherwise anomalous

results will occur due to the fact the direct wave will not be acting into the same microphone sensitivity and will have a greater or lesser effect depending upon orientation.

Second, the nature of the signal to be measured should be known. Possibly it is not known exactly. In either case a good 1/3 octave filter or band pass filter should be used to ensure that the power measured is of the desired spectrum in the event other unwanted noises are present.

Third, the logarithmic amplifier should have a wide band response not only to handle the frequencies involved in the measurement but to handle the decay rates involved. Logarithmic amplifiers, in general, have a 50 or 60 decibel range, the 60 db range being most desirable.

The Memoscope has the potential of measuring a maximum decay rate of 5×10^5 db/sec. This is established on the basis of a maximum sweeprate of 10 microseconds per division for 10 divisions, 50 db vertical displacement and a 45 degree slope for maximum accuracy. Resolution of decay rate to $\pm$ 10% is quite easily accomplished even with little or no experience in use of the reverberation technique and evaluation of decay curves.

Lastly, a true RMS meter similar in type to the John Fluke 910A is recommended. This meter is unique in that it reads true RMS voltage regardless of waveform and has provision for low or high damping. When used with high damping the problem of integation[19] to get a voltage measurement is largely eliminated.

[19] R. W. Young, A Brief Guide to Noise Measurement and Analysis, USNEL, Report 609, May 1955

A possible test setup is shown in Figure 9. The decade attenuator and oscillator are useful in the calibration of the logarithmic amplifier - Memoscope combination.

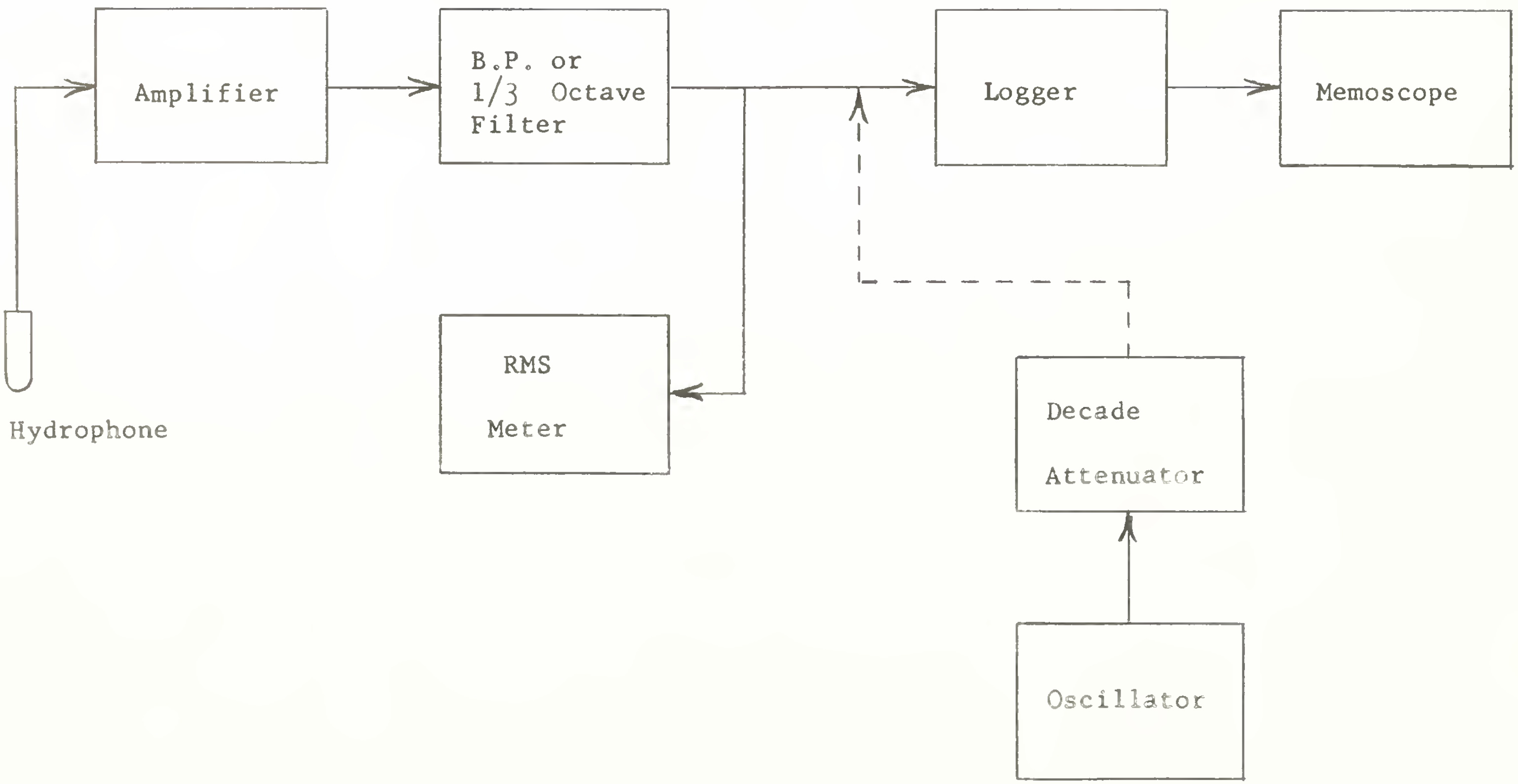


Figure 9 Block Diagram For Proposed Test Setup

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APPENDIX I

Directivity Ratios

Directivity ratios were determined in the following manner since accurate calibration data was available or measured only for certain frequencies. From the beam patterns, horizontal and vertical, the Directivity Ratio was calculated. Since the hydrophone closely approximated a line hydrophone of length ℓ, the equation for the pressure field of a cylindrical line hydrophone is given in NDRC Summary[1] as

$$P = \frac{\sin\left(\frac{k\ell}{2}\sin\gamma\right)}{\frac{k\ell}{2}\sin\gamma}$$

where γ = angle off axis.

Then since

$$D = \frac{P_{ax}^2}{P_{ave}^2} = \frac{1}{\frac{2}{\pi}\int_0^{\pi/2}\left[\frac{\sin\left(\frac{k\ell}{2}\sin\gamma\right)}{\frac{k\ell}{2}\sin\gamma}\right]^2 d\gamma}$$

From Schelkunoff[2] the result of the integration gives

$$D = \frac{x}{\frac{\cos 2x - 1}{2x} + Si\ 2x}$$

where $x = k\ell/2 = \ell/\lambda$

This is then evaluated for various values of ℓ/λ and is presented as Figure I - 1.

From the directivity ratios calculated the curve yielded values of ℓ/λ . This was then used to determine an average length, ℓ' for

[1]The Design and Construction of Magnetostriction Transducers, Summary Technical Report of Division 6 NDRC Vol 13, pp. 113-114, Washington DC, 1946.

[2]S. A. Schelkunoff, Applied Mathematics for Engineers and Scientists, D. Van Nostrand, 1948.

the given hydrophone. Then by use of this ℓ' various ℓ/λ values were calculated for frequencies for which beam patterns were not available. Entering Figure I-1 with ℓ'/λ yields the directivity ratio for the vertical plane. Then by use of the product theorem[3], the horizontal and vertical directivity ratios were multiplied resulting in the directivity ratios given in Table I-1.

Table I-1

Directivity Ratios

Frequency Kc	Serial 207	Serial 204
10	1.07	1.05
15	1.20	1.12
20	1.355	1.25
25	1.49	1.41
30	1.63	1.60
35	1.81	1.60
40	1.99	1.77
45	2.30	2.22
50	2.52	2.52

[3] NDRC Summary, op. cit.

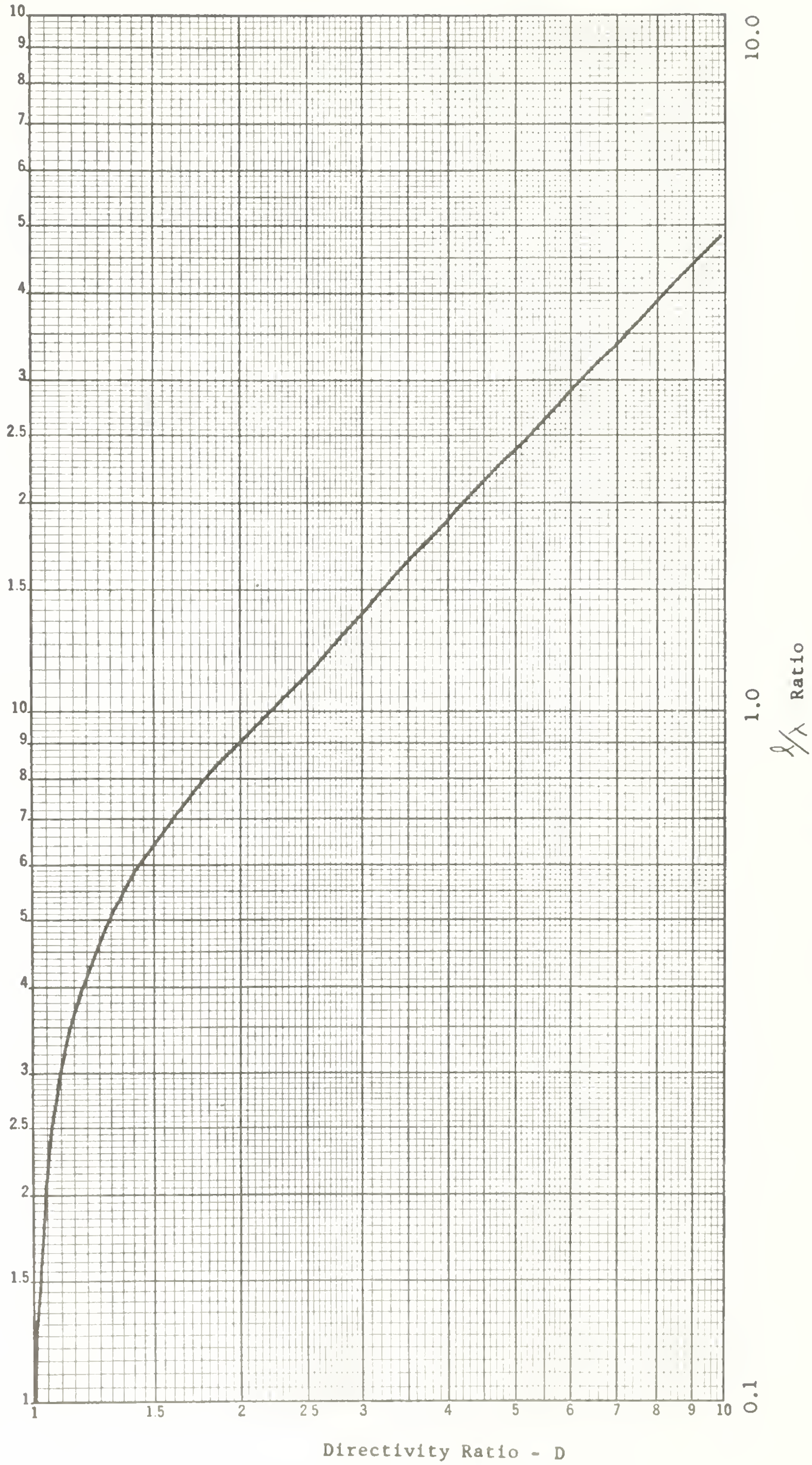


Figure I-1 Graph of Directivity Ratio Vs ℓ/λ

APPENDIX II

Hydrophone Calibration

Table II-1

Open circuit receiving response of LC-32, Serial 204, used as receiving hydrophone obtained by reciprocity calibration using pulse technique.

Frequency Kc	db re 1 volt/μbar
10	-102.47
15	-103.25
20	-104.00
25	-103.7
30	-105.08
35	-105.74
40	-106.3
45	-107.47
50	-107.63

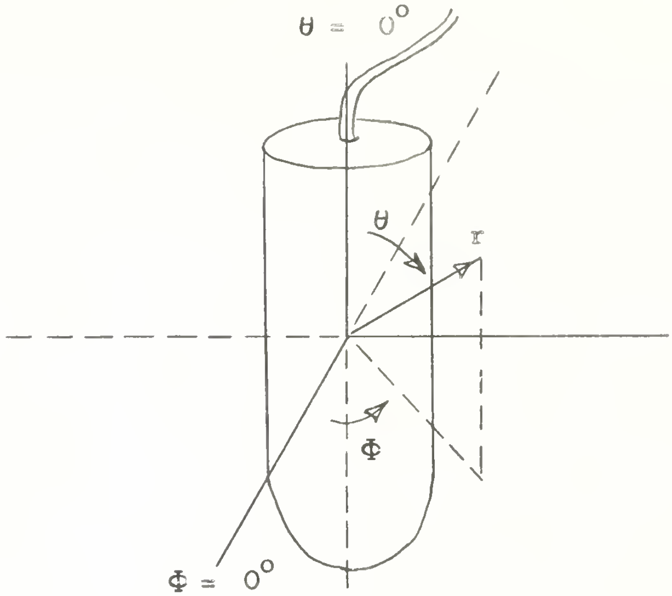
Table II-2

R_p Equivalent parallel resistance of LC-32 serial 207

Frequency Kc	Pulse Kilohms	Reverberations* Kilohms
10	43.4	35.91
15	25.36	20.36
20	15.47	12.41
25	9.75	7.81
30	6.55	5.76
35	4.67	4.12
40	3.47	3.08
45	2.52	2.34
50	1.79	1.78

*R_p for reverberation techniques was obtained by use of a sine wave oscillator with values taken at center frequency, $\pm$ 2.5%, and $\pm$ 5%, then averaged. For example, at 10Kc measurements were made at 10Kc, 9.5Kc, 9.75Kc, 10.25Kc, and 10.5Kc. It should also be commented that under the various tank conditions, i.e. no lining, six blocks, 12 blocks, the observed R_p was within $\pm$ 3% which most likely is due to errors in measurement rather than due to change in R_p. It is expected that R_p would vary under widely different tank conditions, but in the case investigated the variation is not considered significant. Hence, R_p is assumed constant for reverberation technique and is a source of a small error.

TRANSDUCER COORDINATE
DIAGRAM



The spherical coordinate system is used to define the angles in the directivity pattern; θ and Φ shown in the above diagram give the directions in which the response is measured. The transducer is placed in the frame of reference with its axis of symmetry coincident with $\theta = 0^{\circ}$, its fiducial mark in the $\Phi = 0^{\circ}$ plane, and its center at $r = 0$.

The two patterns most frequently measured are:

(1) those made by holding Φ constant at some angle and rotating θ through 360°.

$$(\Phi = a^{\circ};\ \text{rotate } \theta)$$

(2) those made by holding θ constant at some angle and rotating Φ through 360°.

$$(\theta = b^{\circ};\ \text{rotate } \Phi)$$

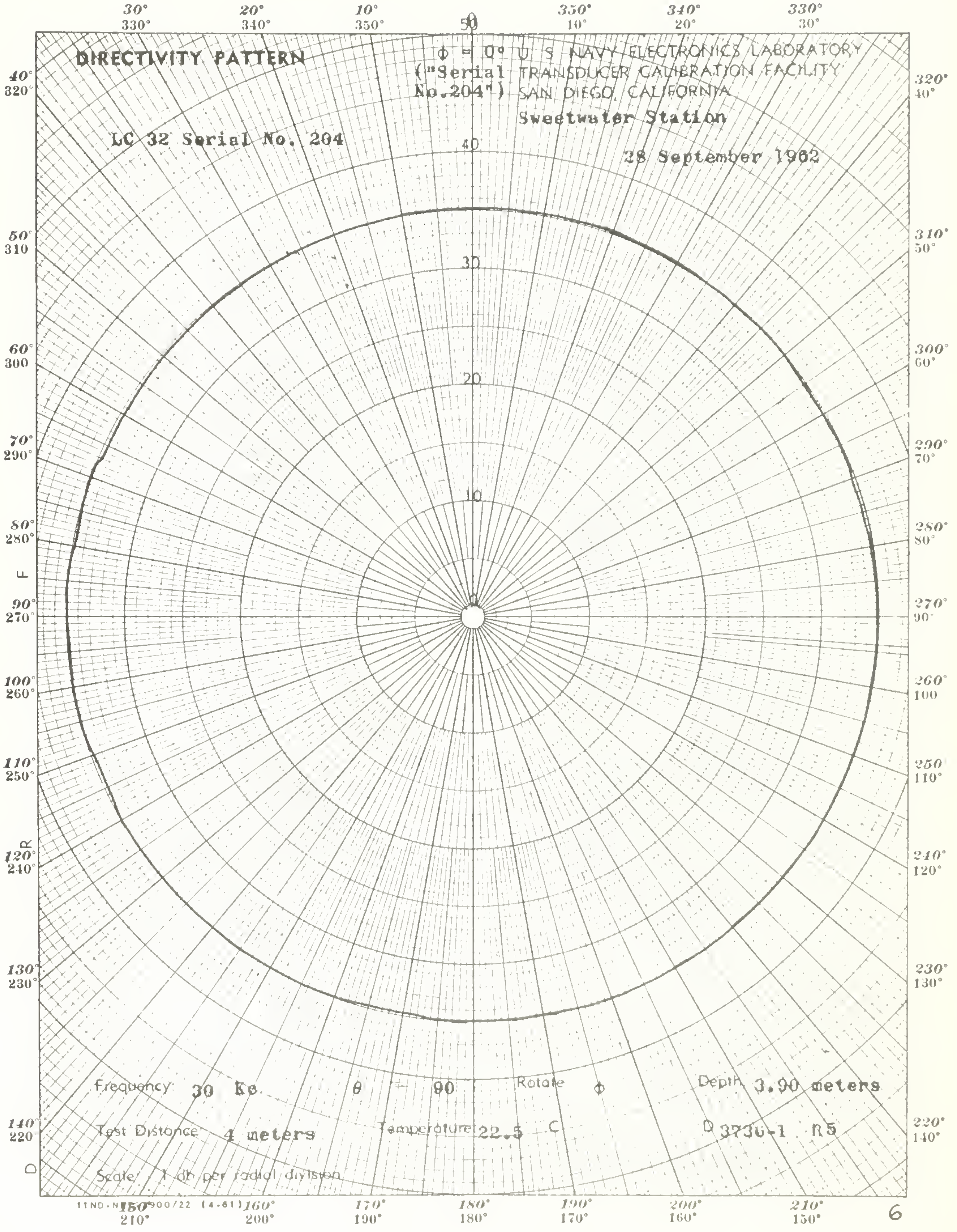


Figure II-1 Horizontal Directivity Pattern, Ser. 204, 30Kc

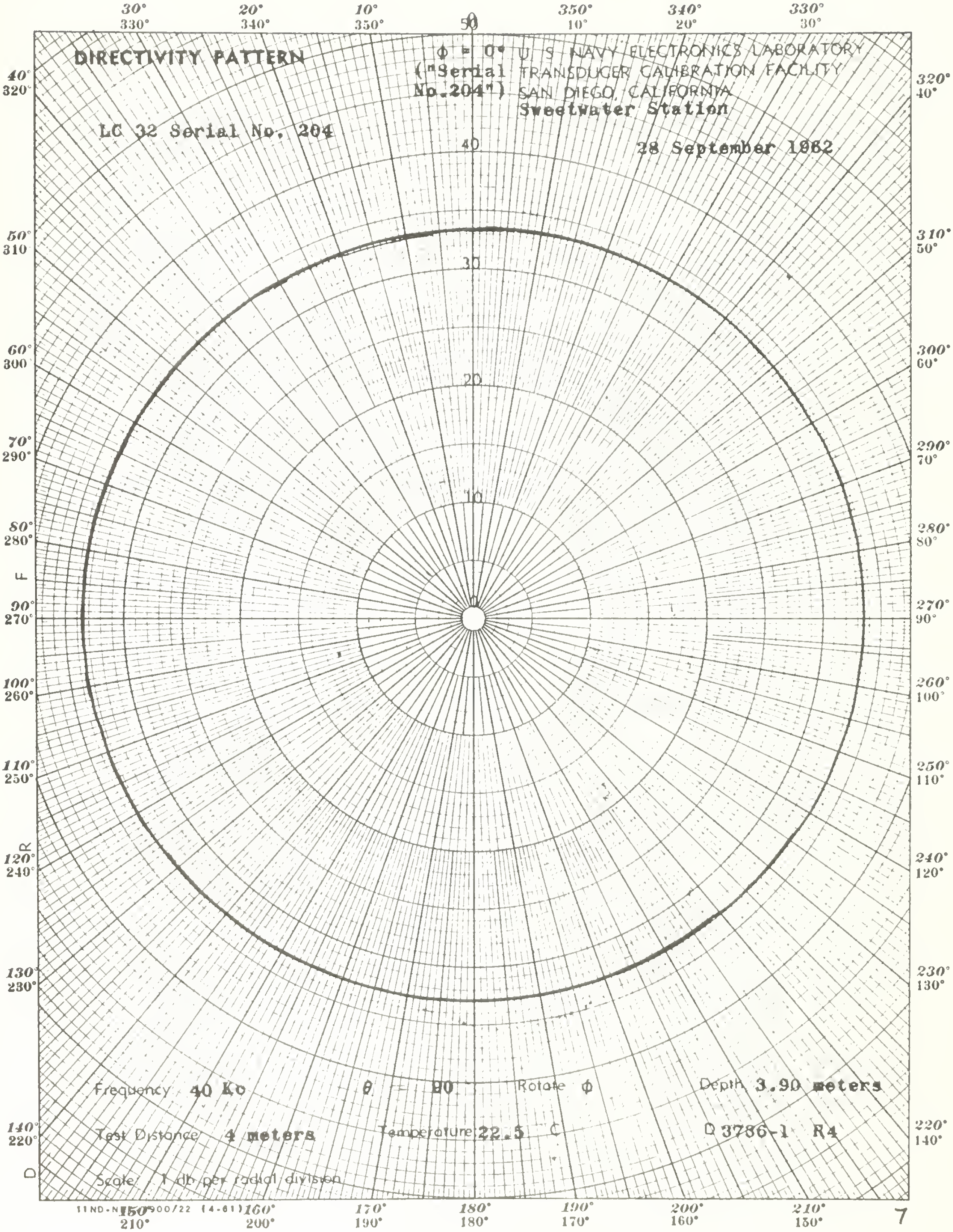


Figure II-2 Horizontal Directivity Pattern, Ser. 204, 40Kc

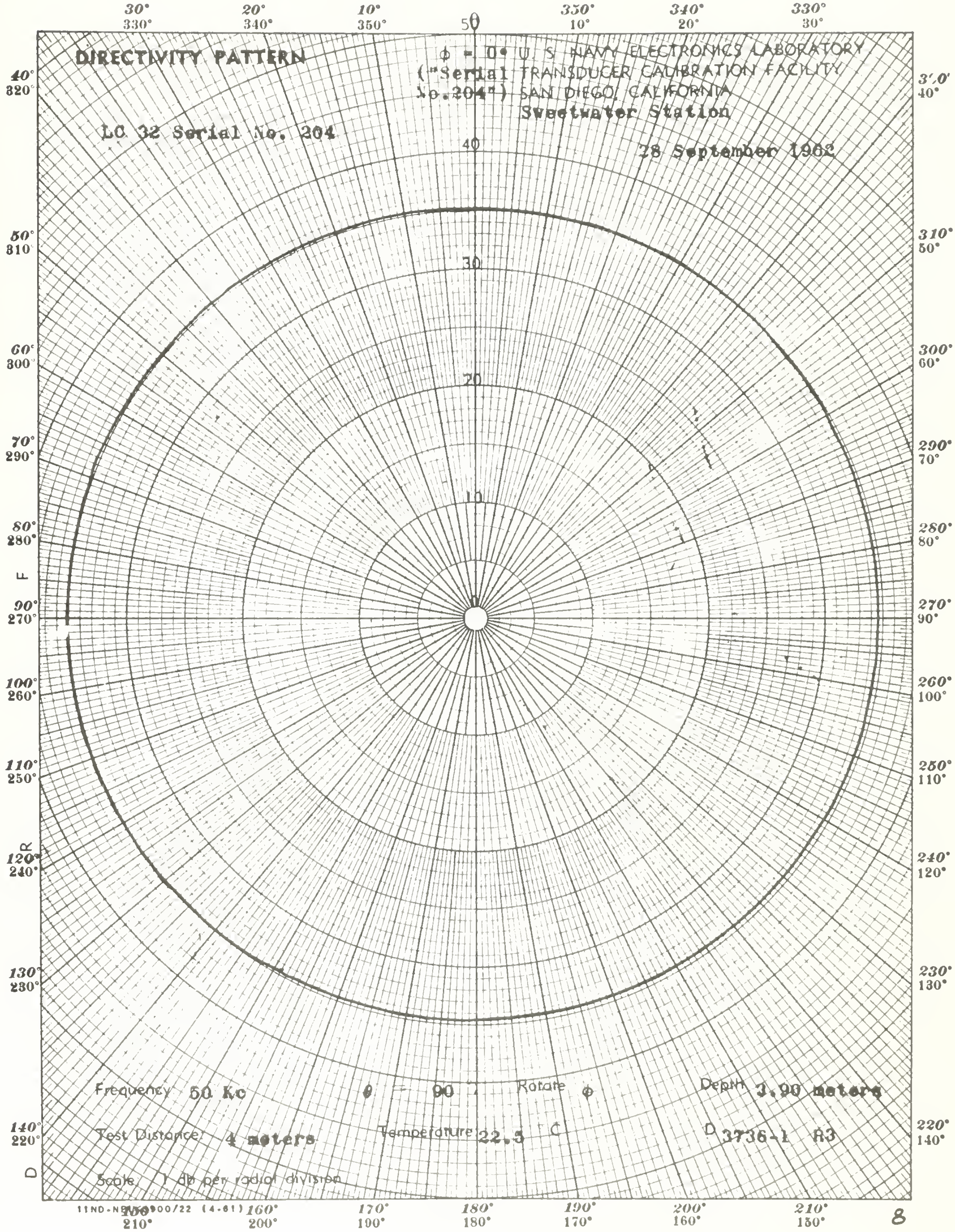


Figure II-3 Horizontal Directivity Pattern, Ser. 204, 50Kc

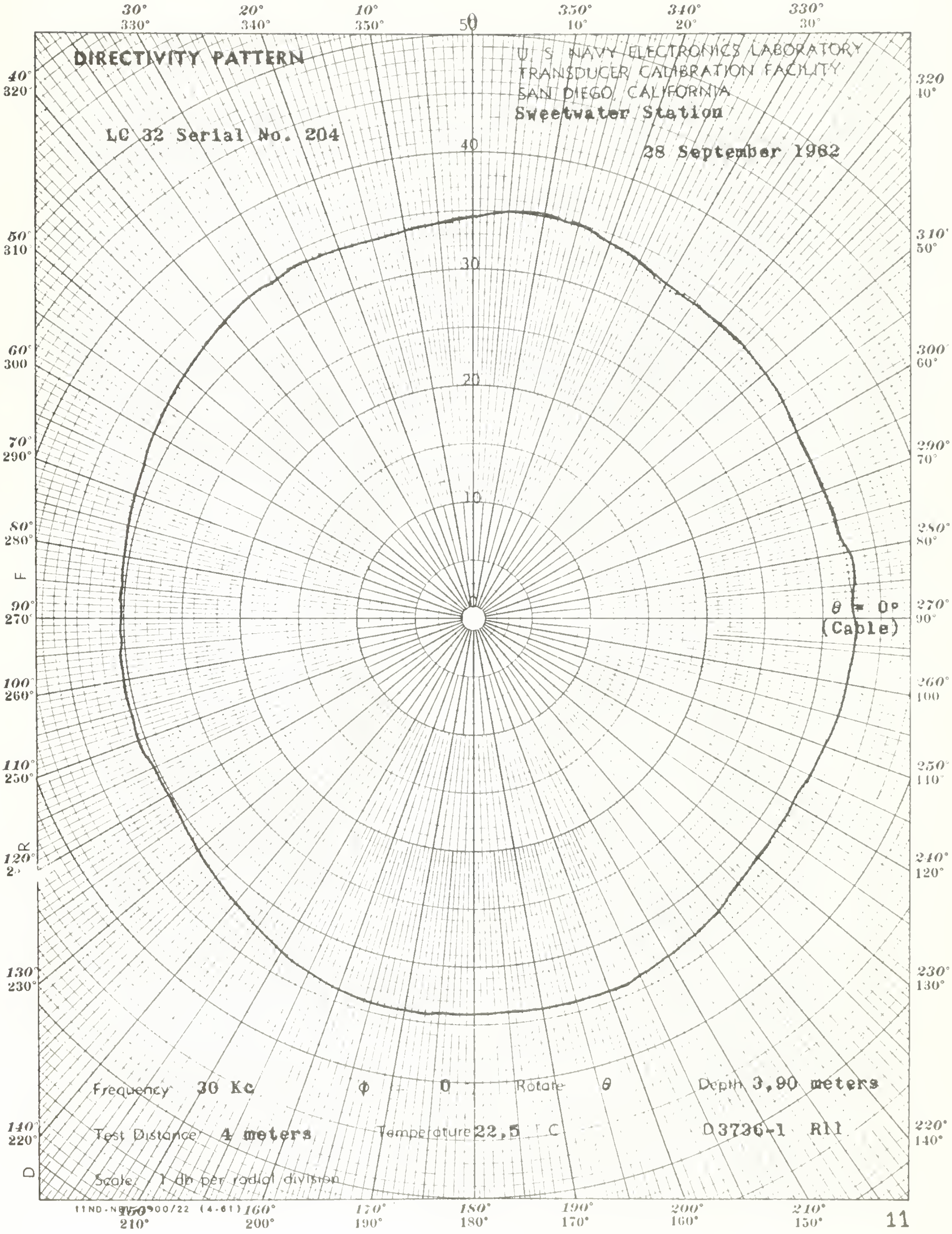


Figure II-4 Vertical Directivity Pattern, Ser. 204, 30Kc

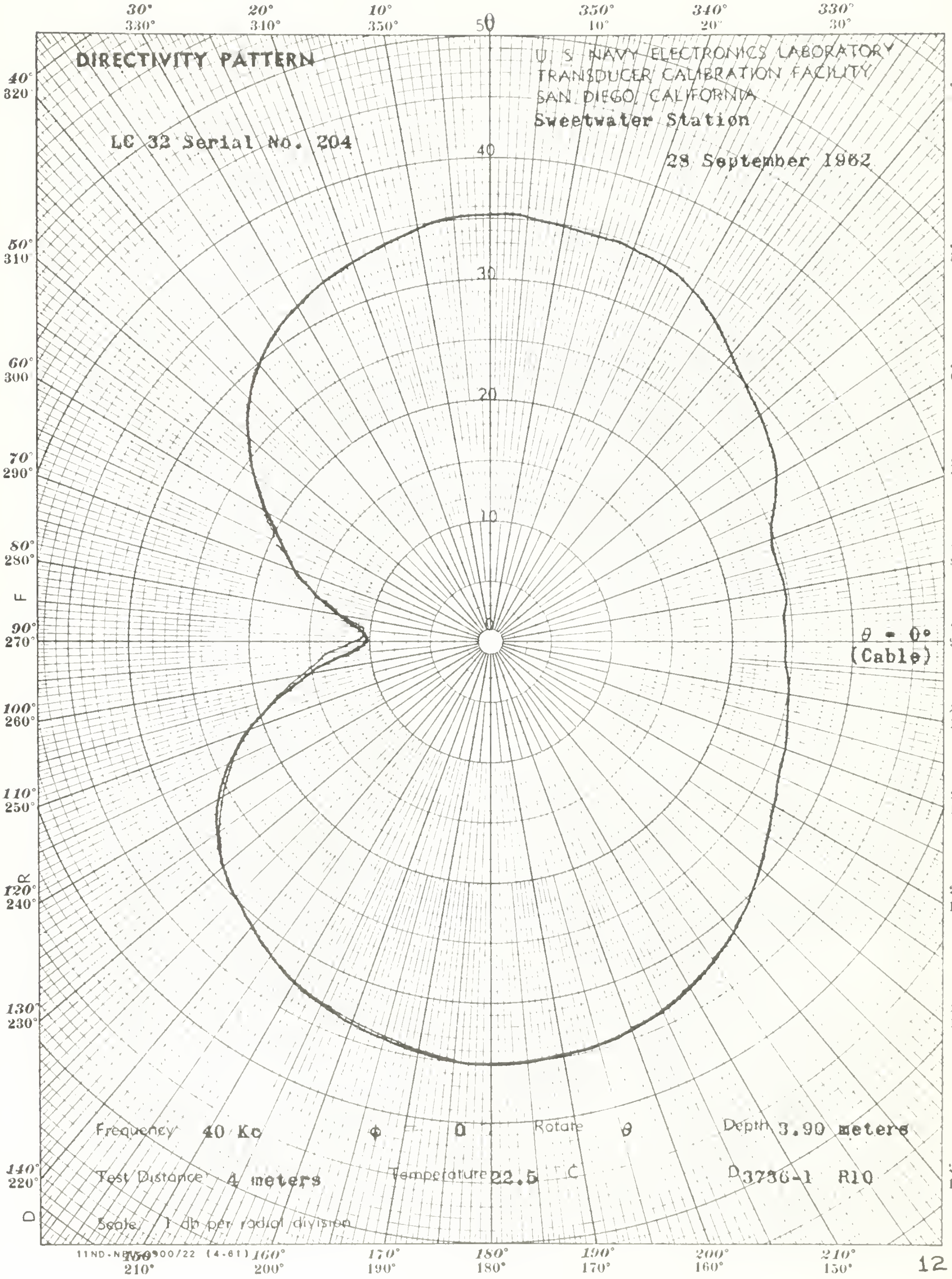


Figure II-5 Vertical Directivity Pattern, Ser. 204, 40Kc

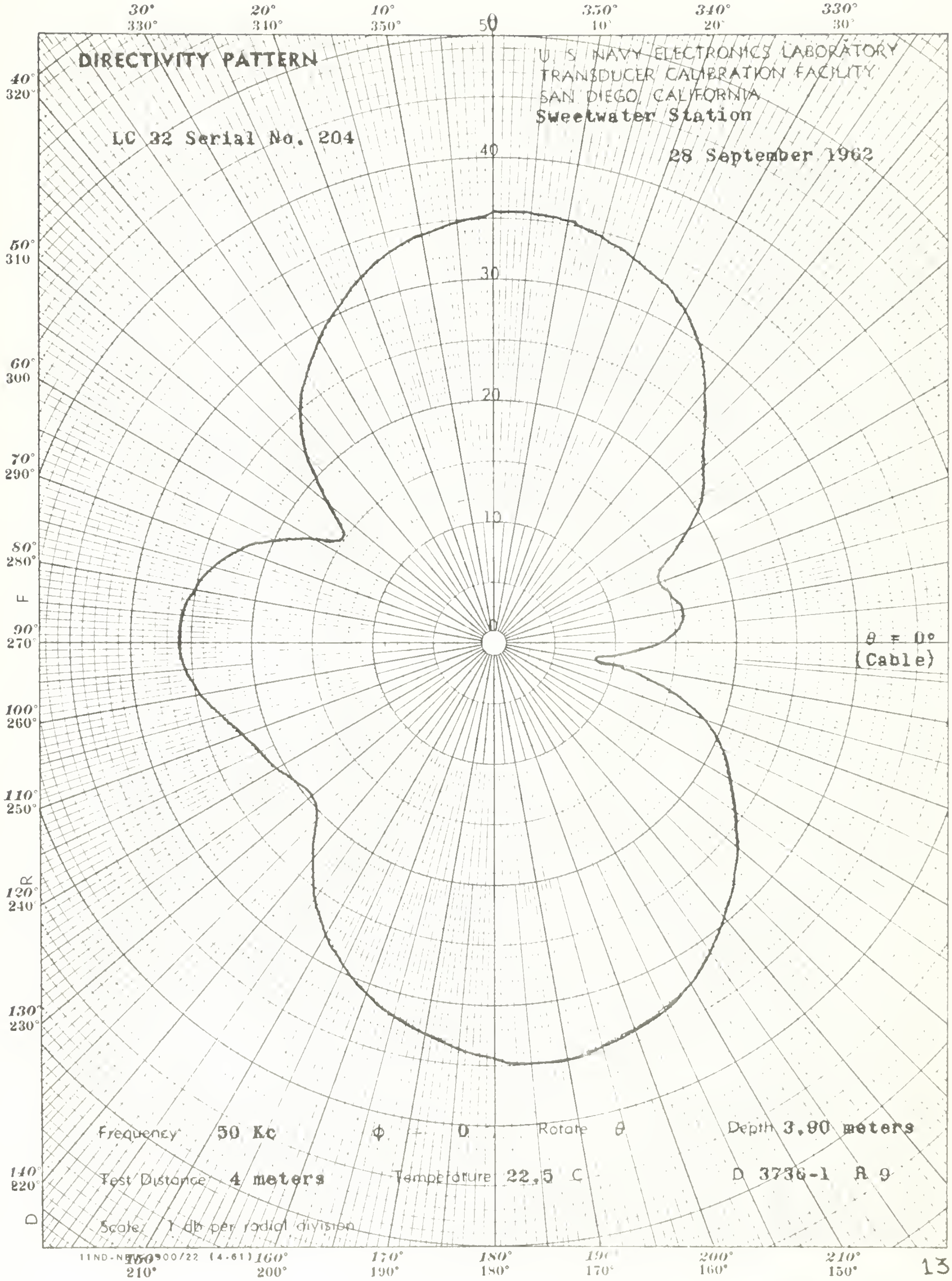


Figure II-6 Vertical Directivity Pattern, Ser. 204, 50Kc

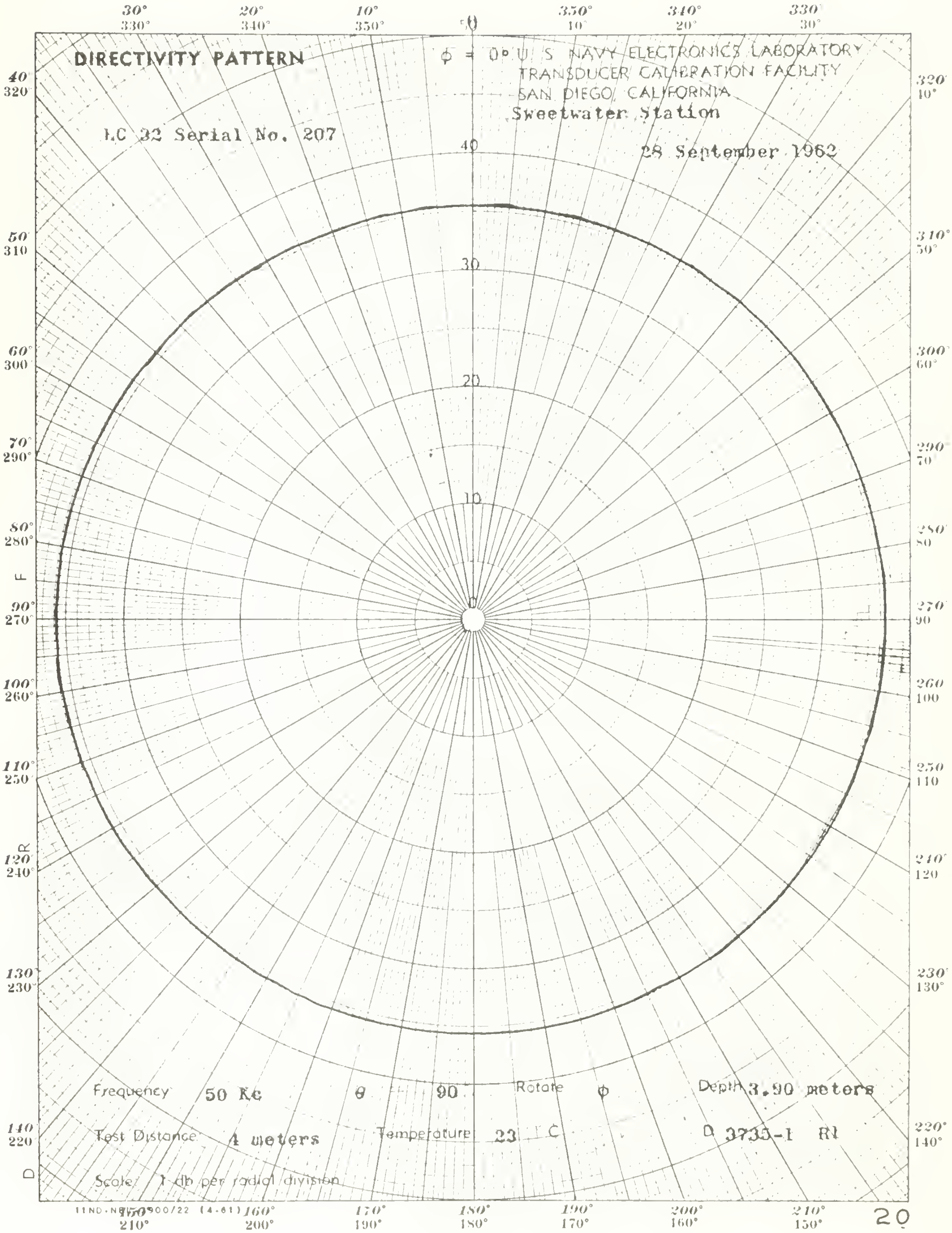


Figure II-7 Horizontal Directivity Pattern, Ser. 207, 50Kc

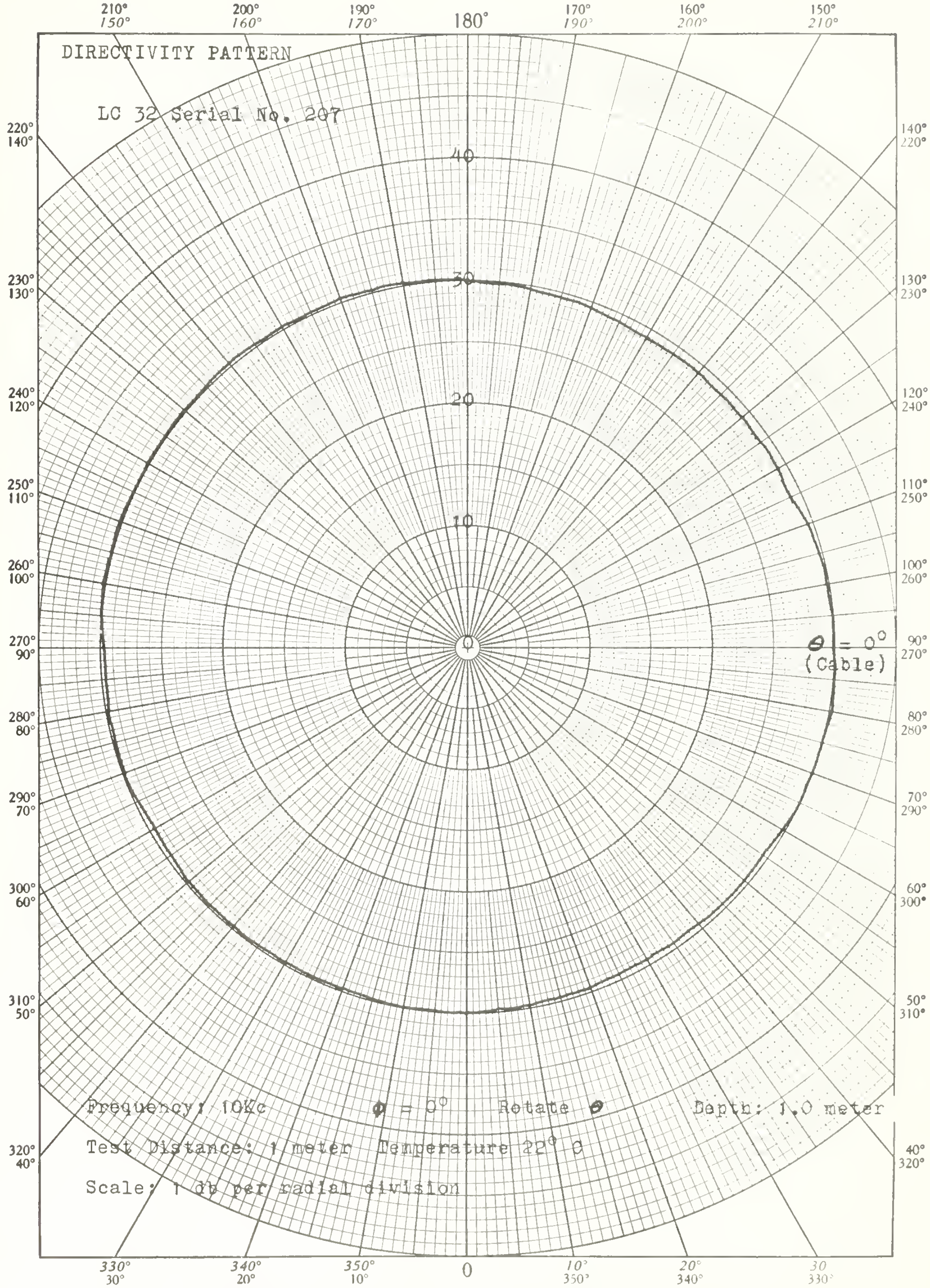


Figure II-8 Vertical Directivity Pattern, Ser. 207, 10Kc

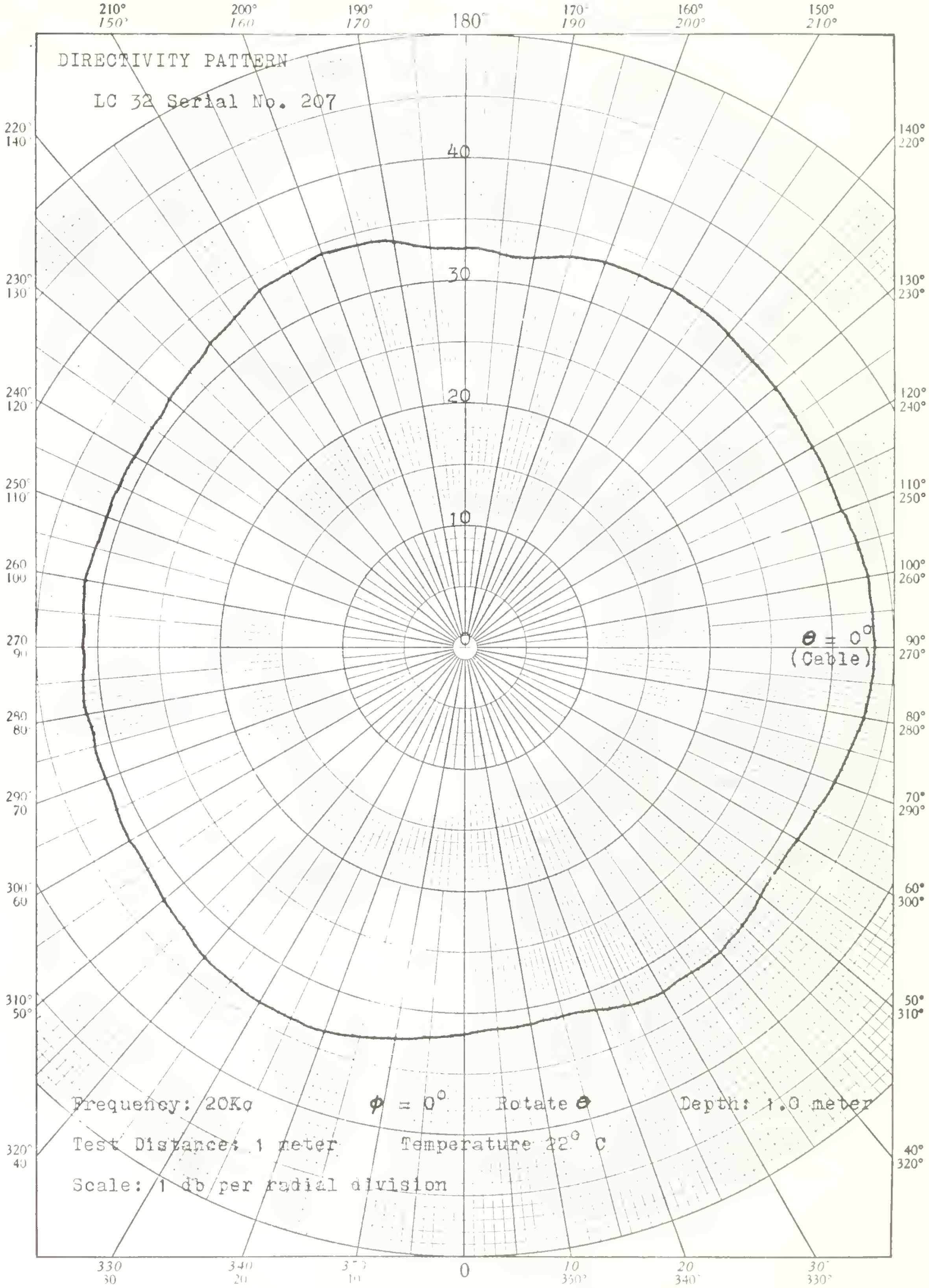


Figure II-9 Vertical Directivity Pattern, Ser. 207, 20Kc

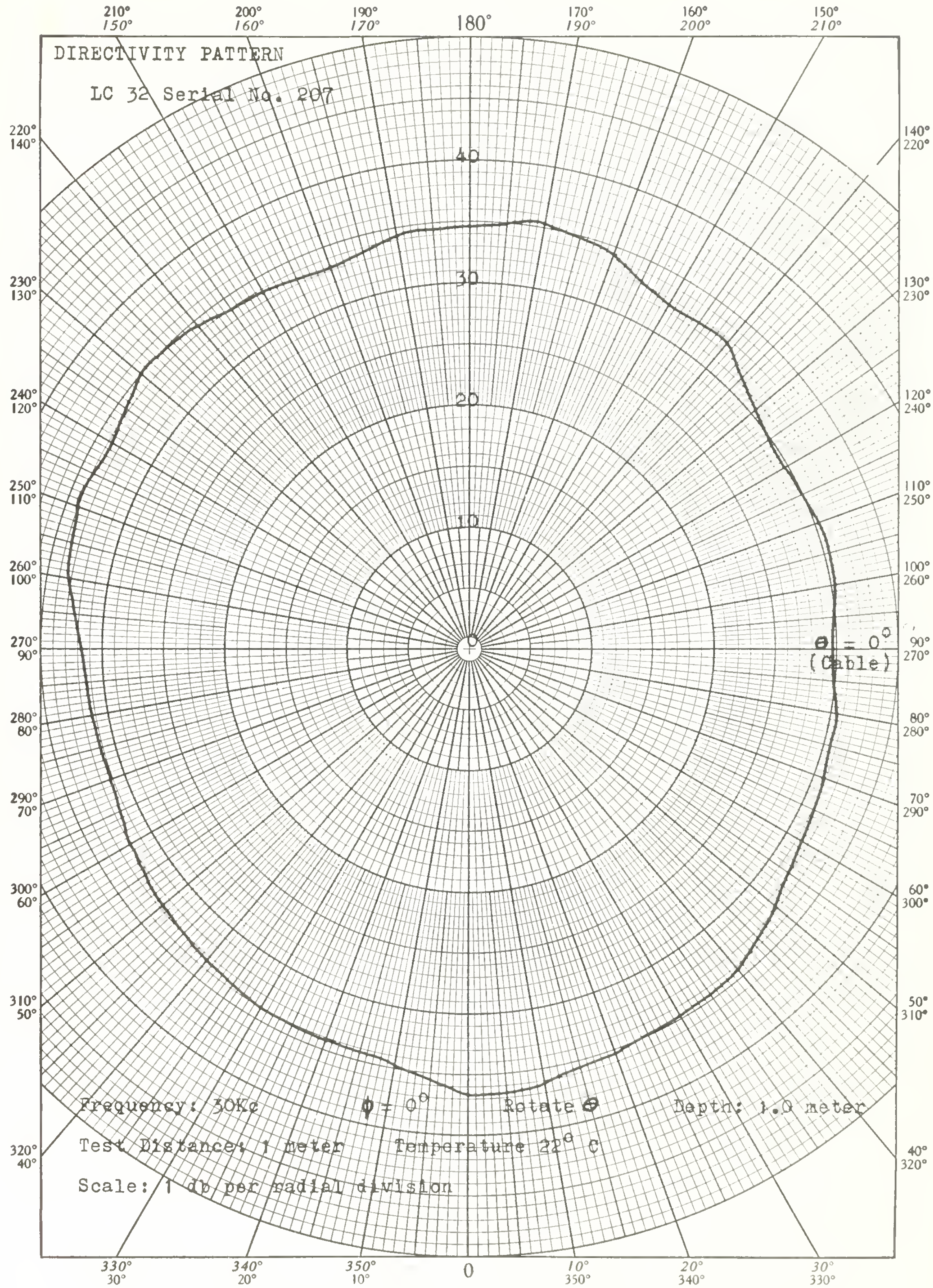


Figure II-10 Vertical Directivity Pattern, Ser. 207, 30Kc

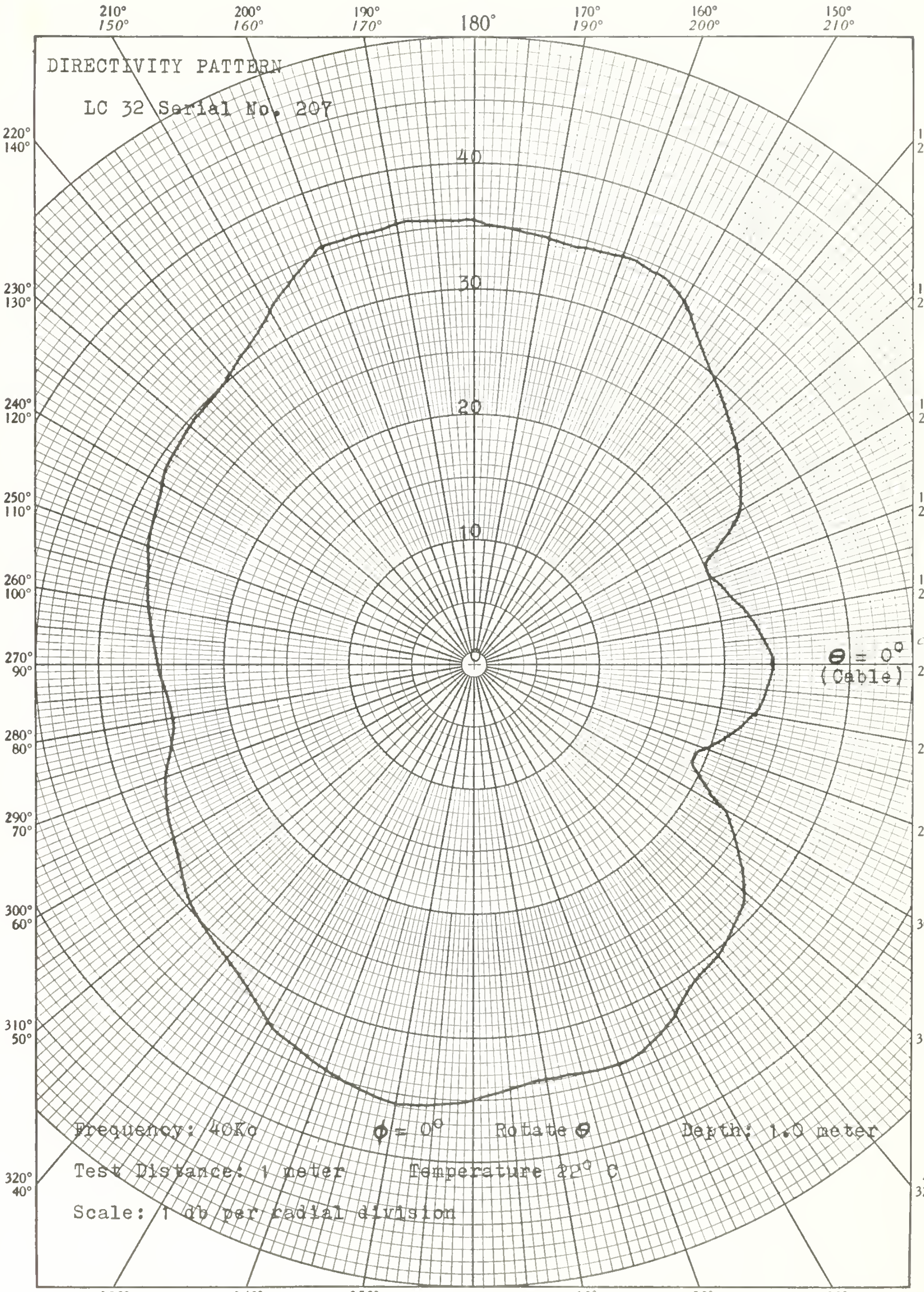


Figure II-11 Vertical Directivity Pattern, Ser. 207, 40Kc

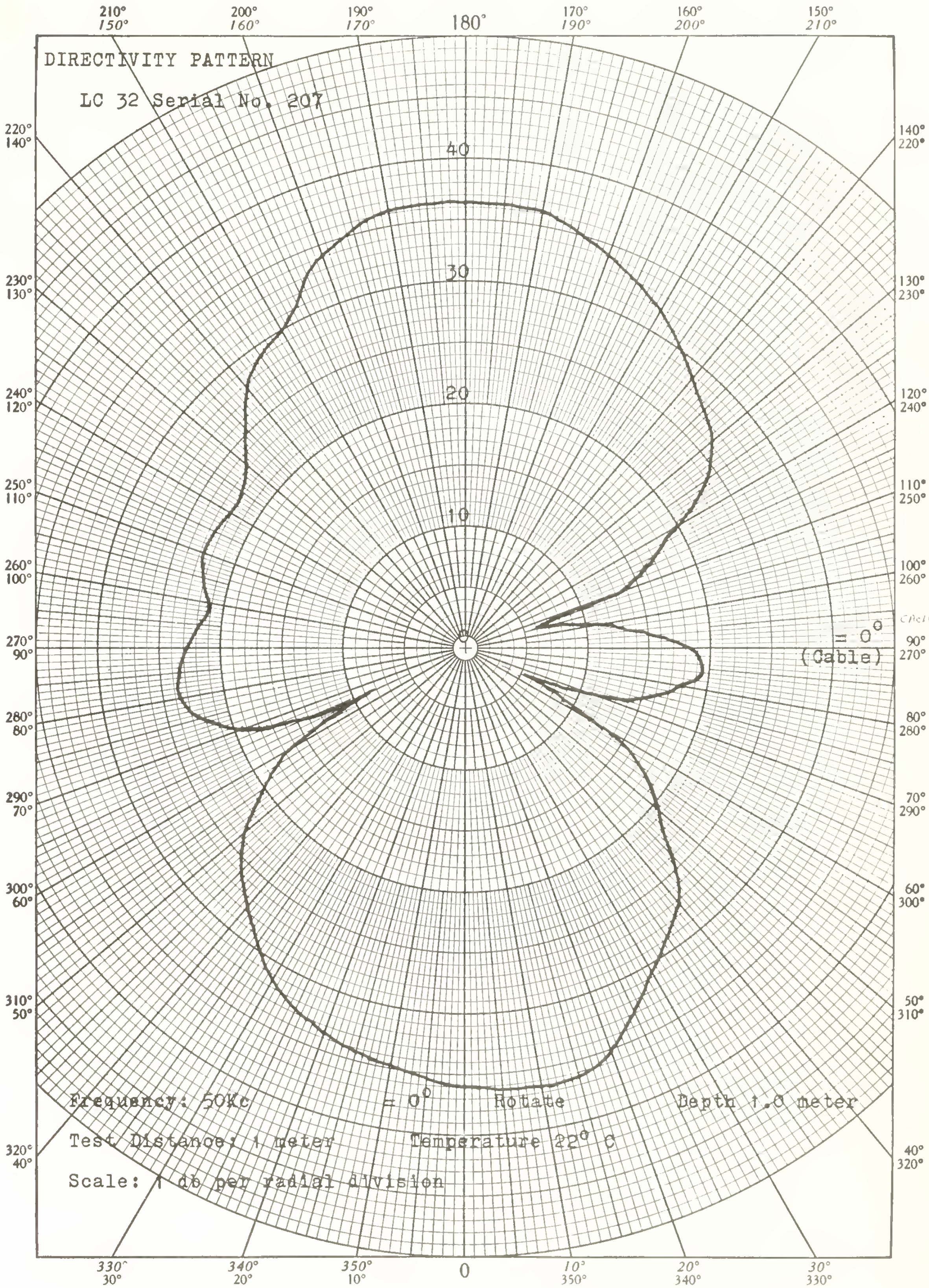


Figure II-12 Vertical Directivity Pattern, Ser. 207, 50Kc